

Calculations (to ANL) for the Flow About a Lens

Tabulate and plot graph of

M and $\frac{dM}{d \cos \theta}$ as a function of $\cos \theta$ in the range

0 (Also, tabulate $\frac{M}{\sin \theta}$ as a function of θ).

The quantity M is given by a contour integral which can be evaluated when $\frac{n}{q}$ is a rational multiple of π .

Let

$\frac{n}{q}$ where n, q are relatively prime integers of opposite

parity (i.e., one is even and one is odd). Introduce the following sums:

$$A = \sum_{j=1}^{q-1} (-1)^{j-1} \csc^3 j$$

$$B = \sum_{j=1}^{q-1} (-1)^{j-1} \csc^3 j \cot j$$

$$C = \sum_{k=1}^{n-1} (-1)^{k-1} \csc^3 k \quad \text{where} \quad \frac{n}{q} = \frac{2}{\dots}$$

$$D = \sum_{k=1}^{n-1} (-1)^{k-1} \csc^3 k$$

$$E = \sum_{k=1}^{n-1} (-1)^{k-1} \csc^3 k \cot k$$

$$F = \sum_{k=1}^{n-1} (-1)^{k-1} \csc^3 k \cot k$$

(1) In terms of these sums,

$$M = M - M$$

where

$$M = \sin^3 A - \frac{q}{2n} \left(1 - \frac{q^2}{n^2} \right) C - \frac{q^3}{n^3} D$$

$$M = \frac{1}{3} (2 - 3 \cos^3 - \cos^3)$$

(2) In terms of these sums,

$$\frac{dM}{d \cos} = \frac{dM}{d \cos} - \frac{dM}{d \cos}$$

where

$$\frac{dM}{d \cos} = \sin^2$$

and

$$\frac{dM}{d \cos} = 3 \sin^2 - \cos^3 \cot M$$

$$B = \frac{q}{720} \left(\frac{q^2}{n^2} - 1 \right) - \frac{7q^2}{n^2} 17$$

$$\frac{q^2}{6n^2} - \frac{3q^2}{n^2} - 1 C - \frac{q^4}{n^4} D$$

$$\frac{q^3}{6n^3} - \frac{q^2}{n^2} - 1 E - \frac{q^5}{n^5} F$$

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(3) For special (limiting) values of :

$$\text{For } = 0 : \quad M = 2 \quad \sum_{j=1} \frac{(-1)^{j-1}}{j^3} - \frac{2}{3} = 1.475$$

$$\frac{dM}{d \cos} = 2 \quad \sum_{j=1} \frac{(-1)^{j-1}}{j^3} - \log_e 2 = 1.309$$

$$\text{For } = \frac{1}{2} : \quad M = \frac{1}{8}$$

$$\frac{dM}{d \cos} = \frac{1}{8}$$

$$\text{For } = : \quad M = 0$$

$$\frac{dM}{d \cos} = 0$$

$$\frac{d^2 M}{(d \cos)^2} =$$

$$\frac{M}{\sin^3} = \frac{4}{3}$$

Care must be taken in these calculations for the following reason. The sums A, B become quite large, with values in the thousands; the factors multiplying C, D, E, F, likewise become very large. The additions and subtractions finally give answers in the units and even in decimals. Thus, perhaps seven or eight figures must be used all along.

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and $\frac{1}{p} \cos \frac{2\pi}{p}$ to be computed for the following values of

n and q ($\frac{n}{p}$ is an integer)

- (a) $n = 1 : q = 4, 6.$
- (b) $n = 2 : q = 5, 5, 7.$
- (c) $n = 3 : q = 4, 8.$
- (d) $n = 4 : q = 5, 7, 9.$
- (e) $n = 7 : q = 8.$
- (f) $n = 15 : q = 16.$

In addition the special values $= 0, \frac{1}{2}$, given above are to be included in the tabulation.

The most important ranges of $\cos$ are:

- (a) $-1.0 \cos - 0.8$
- (b) $0.2 \cos 0.6$

If the distribution of points in the ranges is not smooth enough, you may use your discretion in calculating additional points according to the rule on page 1.

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